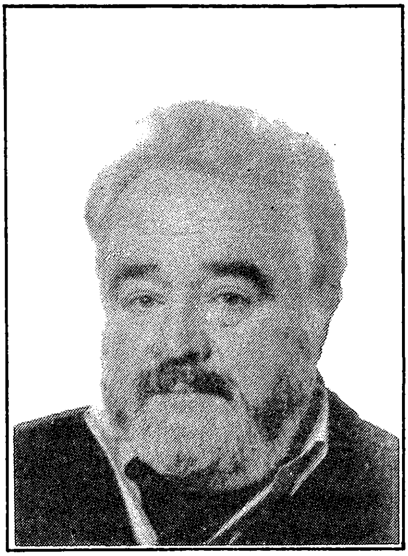

MEDIEVAL THEORETICAL MECHANICS

K.J. Van Camp

For many physicists the history of physics in the middle ages is a black hole. When one talks about the history of physics one usually jumps from Ptolemeus to Copernicus and Galileo Galilei is put forward as a deus ex machina. I knew I had the necessary documentation to elucidate for myself the history of mechanics in the middle ages in West Europe. The Sarton lecture was the ideal opportunity to study this documentation and to distil this very schematic survey.

From the fall of the Roman empire until the year 1000 there was no longer any interest in science in Western Europe. One even can claim that science no longer existed. The Greek knowledge was not lost however. Two sources pulled West-Europe out of its ignorance. The Arab world had translated the bulk of Greek scientific, philosophical, and theological writings. Also other sources such as Hindu mathematics were included in the Arab manuscripts (e.g. the invention of the number zero). On the other hand, Byzantium did still possess many original Greek books and the Greek language was not lost.

The translation Period 1000-1250

One man at the right moment can play a decisive role in history and this is the case for the history of science as well. Gerbert d'Aurillac (946-1003) was the first to make contact with the Arab world. He came across Latin translations about e.g. the astrolabe. He himself was not an original thinker but he was an excellent teacher. He thought in the cathedral-school of Reims and his pupils, in whom the fire for science was kindled, founded new cathedral-schools in Köln, Utrecht, Sens, Cambrai, Chartres, Laon, Auxerre, and Lyon.

The interest in scientific discussion was reborn. We know e.g. of eight letters going back and forth between Köln and Liège about the value of the ratio of the sides of two squares of which one has the double surface of the other. They didn't grasp this ratio was $\sqrt{2}$, but the mere fact that the discussion went on, was a clear sign science was reborn in the West.

To avoid an endless list of names of translators, we will focus on the two most important ones of the Greek science.

Through the reconquista of Spain by the christians arabic knowledge became available to the West. Especially the fall of Toledo in 1085 was a landmark in the translation period. Many translators went to Toledo and in a spirit of mutual comprehension between Christians, Jews, and Arabs the enormous task of re-instructing Western Europe was started. The task was arduous because too often the translators did not grasp the meaning of the manuscripts. Often less important writings were translated instead of the great Greek books. Above this chaotic approach one person did stand out : Gerardo di Cremona (1114-1187). If he had been the only translator he could have kindled the flame of Greek science in the West all by himself. In Toledo he learns the Arab language. Then all great Greek books are translated by him : the Almagest of Ptolemeus, the Physics of Aristotle, the Elements of Euclides, the algebra of Al-Kwarizmi, the medical writings of Galen, etc... Gerardo was "the" translator from Arab to Latin in Spain.

A second source of information was available in the south of Italy, especially in Sicily. Sicily had never lost contact with Byzantium. Many original Greek manuscripts were collected in Sicily during the 12th century. Also here a number of scholars translated from Greek into Latin. The Greek language was lost in West-Europe and only a happy few could manage the translation. The most productive translator was the flemish dominican monk Willem Van Moerbeke (1215-1286). His personal friend Thomas Aquinas complained about the many imprecise translations and urged Willem Van Moerbeke to make good Latin texts available to the West. He especially concentrated his efforts on Archimedes and Aristotle.

His transcripts of Archimedes reached the renaissance period and were the first ever to be printed (Venice 1503).

The western medieval theorical mechanics

When we talk about the history of mechanics we mean the theorical (mathematical) mechanics. Another mechanics or better statics was at its very summit. The statics applied in the construction of the gothic cathedrals were indeed very daring, but this is another story.

We will divide mechanics in its three major components which are statics, kinematics, and dynamics.

A. Statics

The Greek physicists had a quite correct picture about the equilibrium of mechanical constructions. To my opinion this was due mainly to the fact that they had the daily experience of the scale, both with equal and unequal arms. They had the notion of centre of gravity, moment of a force and even a crude version of the theorem of virtual work. Also the theory of levers was one of the points of excellence of Archimedes. The legend goes that he said *give me a fixed point and I will move the earth.* To the medieval scholars also a good translation of a treatise on scales by Euclides was available. Concerning statics the scholastics had a firm basis through the Greek tractates and did in fact not expand much that knowledge.

B. Kinematics

More exciting is the history of kinematics. Kinematics is the study of motion, speed and acceleration without further investigation of the underlying cause of motion. The search and study of these causes are indeed the domain of dynamics. In the Greek science the kinematics was an integral part of dynamics. Progress was made in kinematics as the medieval scholars separated kinematics from the faulty Greek ideas about dynamics. The first to do so was Geraard van Brussel (Gerardus of

Brussels) in the first half of the 13th century. In his tractate *Liber de motu* he speaks about the evaluation of velocities of points, lines and surfaces in both cases of translation and rotation. The concept of acceleration is unknown to him. He is the first to evaluate a velocity as being proportional (not equal !) to distance covered in a given time. The texts are very naive and to illustrate this I reproduce here the *suppositiones* of book II of the *Liber de Motu* as translated by Clagett : (Clagett 1959).

Suppositions
1. *Of equal squares, the one whose sides are moved more quickly is said to be moved more quickly.*
2. *The one whose sides are moved less quickly is said to be moved less quickly.*
3. *The one whose sides are not moved more quickly is not moved more quickly.*
4. *The one whose sides are not moved less quickly is not moved less quickly.*

All mathematical proofs are based on Euclidian geometry. This in fact will stay as such until the *Principia* of Isaac Newton. Only afterwards get things much easier by the use of algebra and calculus. Both techniques where known by Newton. He indeed invented (together with Leibnitz) differential calculus but according to some historians he did not use it in his *Principia* in order to make it not too easy on the readers so that he could stay ahead of them. Newton was known for his bad character and temper !

To understand the evolution of science from the thirteenth century on, we must keep in mind the invention of the University. The university was a typical West-European institution invented around the year 1200. It was a combination of a science centre and an educational institution and has in fact not drastically changed since then. The first universities where Paris, Bologna, and Oxford. Physics was a part of the fine arts curriculum. The teaching was based mainly on reading the Greek translations of Aristotle and others. The texts where thoroughly discussed through debating on the "questiones". From E. Grant (Grant 1977), I reproduce here some "questiones" on the Physics of Aristotle by Albert of Saxony (1316-1390) used at the university of Paris.

- Whether the existence of a vacuum is possible.
- Whether, if a vacuum existed, a heavy body could move in it.
- Whether something could be moved in a vacuum — if one existed — with a finite velocity.

The next step in kinematics originated from the Merton college at the university of Oxford. In the period from about 1330 to 1350 four masters, namely Thomas Bradwardine, William Heytesburg, Richard Swinehead, and John Dumbleton, will make major contributions to kinematics and dynamics. In their approach to kinematics one can distinct three essentials.

- There is a clear separation between kinematics and dynamics
- Clear definitions of uniform and average speed are put forward. The definition of instant velocity on the other hand is circular !
- A correct definition of uniform accelerated motion is given and the famous *average speed theorema* is formulated for the first time and proven for the case of the uniform accelerated motion. This theorem can be formulated in a modern algebraic way as :

$$s = \langle v \rangle t$$

Note that we have no longer a proportion but an equality. The progress in comparison with Geraard van Brussel is enormous but let us remind we are at least a hundred years further in time. In order to get even a better idea about the progress I give here some *suppositiones* on the uniform motion as proposed by Bradwardine (in translation by Clagett).

Suppositions

1. *Every body. surface, line, and point can be moved uniformly and continually.*
2. *In the case of two local motions which are continued in the same or equal times, the velocities and distances traversed by these (movements) are proportional, i.e., as one velocity is to the other, so the space traversed by the one is to the space traversed by the other.*
3. *In the case of two local motions traversing the same or equal spaces, the velocities are inversely proportional to the times, i.e., as the first velocity is to the second, so the time of the second velocity is to the time of the first.*
4. *A given moving body can be moved with any whatsoever quickness or slowness or a given space can be traversed by any body at all.*

The decades around the middle of the fourteenth century constitute the golden age of the theoretical mechanics in the middle ages. Also in Paris significant progress in kinematics is made around that time. Nicole Oresme, pupil of Jean Buridan, whom we will meet later on in dynamics, introduces a very original two-dimensional diagram. By putting time horizontally and velocity vertically he is able to study different kinds of motion. His insight is so profound that he also grasps that the surface in such a diagram represents the distance covered in a given time, really a remarkable achievement. There exists in literature some dispute on the priority claim of this invention. Also Giovanni di Casali is mentioned in connection to this velocity-time diagram. We illustrate the use of this diagram by proving *the average speed theorema* using Oresme's diagram. The original text by Oresme in his book *De configurationibus qualitatum* is so incomprehensibly difficult, long and tedious that I present here a modernized version of his proof on his original drawing (see also reproduction fig. 2). Let BEC represent the speed as function of time in the lapses of time BA. (Fig. 1). The average speed during this lapses of time is given by DE so that BD = DA. The surface of the triangle ABC and the rectangle ABGF are equal as the small triangles BGE and EFC are identical (24th proposition of first book of Euclides). This means thus that both motions covered the same distances during the lapse of time BA.

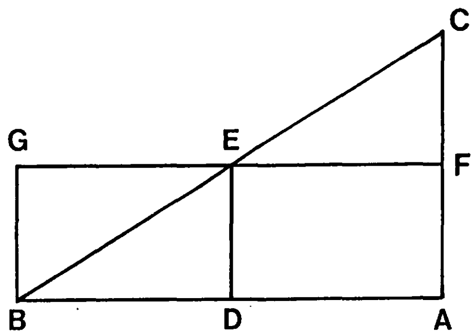

Fig. 1 : Modern drawing for the proof of the average speed theorema by Oresme.

After this stupendous explosion of knowledge within a few decades the advance in kinematics is very meagre. They keep on commenting and discussing and, I personally think that the total absence of experiments and the lack of more convenient mathematical tools made progress very difficult. It took a giant such as Galilei Galileo to make another big leap forward. Using techniques such as the velocity-time diagram of Oresme (see Fig.3) and his own experimental findings in kinematics, he was able to prove the parabolic trajectory of projectiles. He was the first to combine in a correct way experiments, geometry, and physical insight making him the pioneer of modern physics. But let us remind that he did use the findings of medieval science. He was no deus ex machina !

C. Dynamics

The medieval knowledge of dynamics is based upon the misconceptions of the dynamics of Aristotle. It takes the whole middle ages before they grasp that the axioms from which Aristotle starts are totally wrong. Although we study medieval theoretical mechanics it is an absolute must to review the physics of Aristotle to grasp the medieval evolution.

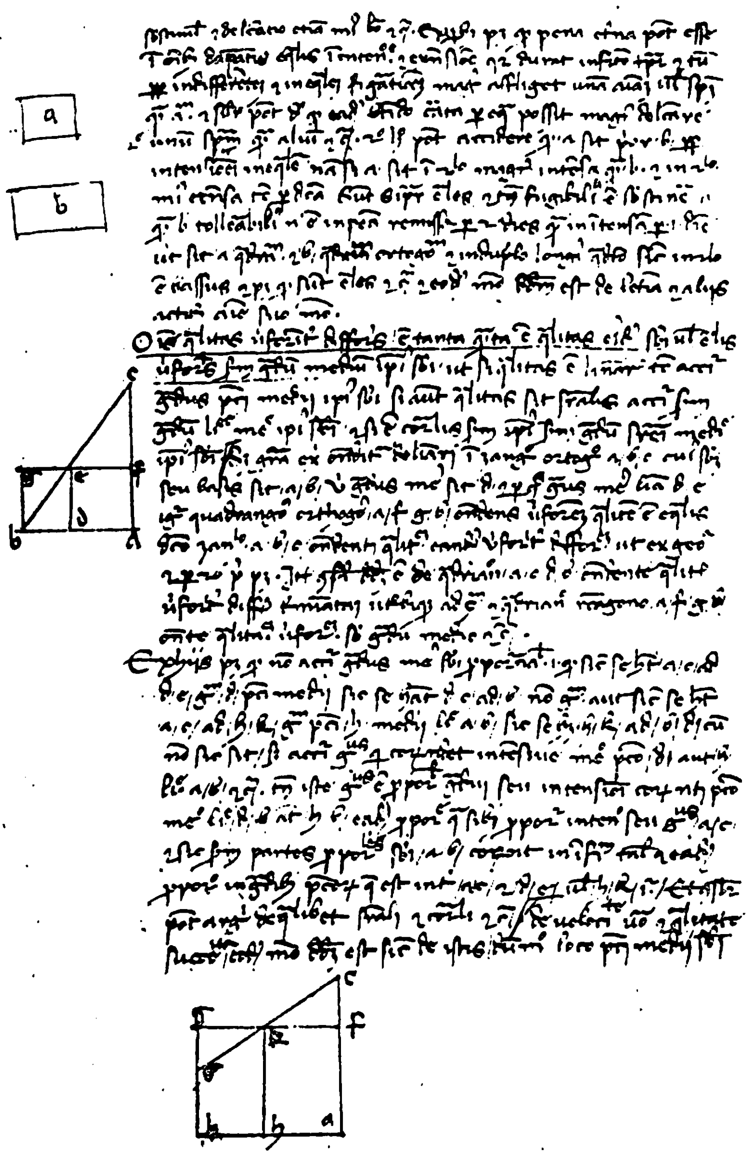

Fig. 2 : A page from a copy of Oresme's *De configurationibus qualitum* with the figures on the average speed theorema (from Clagett 1959).

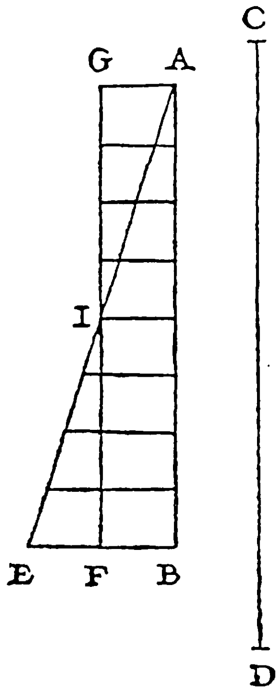

Fig. 3 : Version of figure on the average speed theorema as used by Galileo (from Dover 1954).

Aristotelian Dynamics

Aristotle put forward two different physics. In the celestial mechanics the natural motion is the uniformly circular motion. Everything in the higher spheres is eternal and made out of the quintessence. In our sublunar world, on the other hand, nothing is eternal. The sublunar world is made out of four elements : earth, water, air, and fire. Each element has his own sphere, the earth in the middle and then on more outward shells water and air, and in the outer region fire.

The fall of heavy objects and the rise of flames does not need further explanation in this theory as it is the *natural* motion towards its own sphere. Motion could also be induced by changing the ratios of the four elements in a body by e.g. heating it. In contrast to the "natural motion", one has also the *violent* or *unnatural motion* of e.g. projectiles in their upward trajectory. The axiom which caused the greatest difficulties stated that for any motion there is always the need of a *motor* or *force*. If we inspect more closely the relation between the force **F**, the friction or resistance **R**, the distance **s** and time **t**, the physics of Aristotle was grounded on the following proportional relations (no equalities !)

$$s \approx (F/R)t \qquad\qquad v \approx (F/R)$$

This very ungreek way of presenting this physics by algebraic formulas is certainly an oversimplification. Aristotle himself commented on the relation $v \approx F/R$ by saying that a body could only be moved if the motor or driving force **F** exceeded a minimal value. Certainly a necessary condition otherwise any person could move any object of unlimited size ! The nature of **F** and **R** could differ according to the kind of motion. In the downward motion weight was the motor and the air caused the resistance.

In the violent upward motions both roles were reversed. The weight was the resistance and the air was the source of the motor.

The nightmare of Greek and medieval mechanics was the void. They realized that the acceptance of the above mentioned laws resulted in enormous contradictions for the motion in the void. For the downward motion there was no friction so the speed had to be infinite. On the other hand, no upward motion was possible through the lack of air which excluded any *motor* necessary to move the object upwards. They cut the Gordian knot by stating that the void could not exist !

The medieval dynamics

The history of medieval dynamics is one long battle against the faulty Aristotelean ideas. They try to find a way out of this labyrinth of contradictions by proposing special solutions for special cases. These special cases can be summarized to a certain extend by considering three kinds of motion : a) pushing and pulling, b) throwing, c) falling. We will highlight now the most remarkable theories for the three motions proposed.

a) *Pushing and pulling*

In the medieval period the most accepted theory on the relation between the motive force **F**, the resistance **R**, and the resulting speed **v**, was the one proposed around 1330 by Thomas Bradwardine of Merton College. He tried to reconcile two statements of Aristotle :
1) There is a relation between **v** and the ratio **F/R**; and
2) **F** has to have a minimal value before motion can occur.
The very original solution Bradwardine invented can be formulated in modern terms in the following way.

$$\frac{F_2}{R_2} = \left(\frac{F_1}{R_1}\right)^{v_2/v_1} \quad \textit{with} \quad F_1/R_1 > 1$$

It is clear that for the case $\mathbf{v_2 = 0}$ one has $\mathbf{F_2 = R_2}$, so that $\mathbf{F_2 > R_2}$ is requested in order to have motion. This was a very ingenious invention and it is a pity that experimental control was never made until Galileo Galilei. Although it was ingenious and in accordance with the two remarks of Aristotle, it was totally wrong ! The texts of Bradwardine are for modern physicists exceedingly difficult to read and often quite obscure. Let us remind that the algebraic method we used in stating that law did not exist in the middle ages ! The theory of Bradwardine spread through the whole of Europe and was often copied, commented, and amended. The young Galileo Galilei knew and accepted also this law but later on, through experimenting, proved it wrong and stated an axiom which later on Newton introduced as his first axiom.

b) *Throwing*

The violent motion in the Aristotelean mechanics was explained by giving the air the role of "motor" to propel the stone or the arrow upwards. In the middle ages, again in the middle of the 14th century, three families of more elaborate theories were developed to explain the violent motion.

Franciscus de Marchia around 1320 distributes the propulsion over the medium (air) and the object. The force left behind in the object by the hand or the bow is a temporary motor which dissipates as heat trickles away from hot objects (this comparison is from de Marchia himself). That temporary force is not specified but in a way it is an innovating concept which later on will culminate in the impetus theory of Jean Buridan.

A second set of theories has as main defender William of Ockham (1285-1349). This theory circumvents the difficulties by stating that motion does not exists ! Motion in their minds is a succession of different fixed positions which does not call for further explanation. I think this theory does not deserve further attention ! William of Ockham will be remembered for *Ockhams's razor* which states that of two theories equally in accordance of all observed facts, the theory needing the fewer or simpler axioms is the best. Still a very sound rule !

The third theory is the remarkable impetus theory of Jean Buridan (1295-1358). He was rector of the university of Paris and teacher to Nicole Oresme. Through a very sound reasoning the theories of Franciscus de Marchia and William of Ockham are proved to be wrong. He draws e.g. our attention to the fact that a spear with sharp points on either end reaches the same distance as an ordinary spear. Also the fact that a wheel once it rotates, keeps on turning although it has no rear end on which the air can push, proves something is wrong with the previous proposed explanations. He replaced those theories by the impetus theory in which the motor is totally imbedded into the moving body. Many historians did analyze the impetus theory and the appreciation of the impetus concept ranges from totally wrong to excellent. I myself was able to analyze the English translation by Clagett (1959) of Jean Buridan's original texts. As a physicist I have a very high esteem for Buridan. In my point of view it was qualitatively correct but it used a technical language which in modern physics has other meanings. When he states that the impetus is a force, he is in modern terms speaking wrong. But if one accepts the word force as a synonym of motor and cause of motion, everything is quite right. I think most historians did not know enough physics to see through the archaic vocabulary the exact meanings and ideas. This is enough laudation, let us get down to the facts !

The translation (by Clagett, 1959) from the original latin manuscripts states : *"Therefore, it seems to me that it ought to be said that the motor in moving a moving body impresses in it a certain impetus or a certain motive force on the moving body, [which impetus acts] in the direction towards which the mover was moving the moving body, either up or down, or laterally, or circularly"*. Further on he states that the impetus is directly proportional to the speed v of the mover and the weight of the body. This impetus is according to Buridan permanent and can only be reduced by opposing resistances. He concludes also that without any resistance e.g. a spinning wheel will keep on turning forever. I personally see in those statements a clear and correct phrasing of the modern concept of conservation of linear and angular momentum. Even more astonishing is this description of the dynamics of falling objects. From Clagett (1959) we quote : *"For from the beginning only the gravity was moving it. Therefore, it moved more slowly, but in moving it impressed in the heavy body an impetus. This impetus now [acting] together with its gravity moves it. Therefore, the motion becomes faster; and by the amount it is faster, so the impetus becomes more intense. Therefore, the movement evidently becomes continually faster."* To my opinion as a physicist this is in perfect accordance with the integral version of the second law of Newton

$$\int_0^t G \, dt = \Delta \, mv$$

in which G is the weight, m the mass and v the acquired speed of the falling object. The only uncertaintly in this text is whether one has to integrate over the time (correct) or the distance (way). Jean Buridan was really the most advanced theoretical physicist on mechanics in his time.

It is dramatic that his theory was not backed up by well designed experiments. If this would have been the case evolution in physics could have gained a few hundred years. Now we had to wait until 1687 for the publication of the *principia* by Isaac Newton.

Later on his pupil Nicole Oresme will mutilate this excellent theory by denying the permanent character of the impetus. Oresme favoured more the idea of Franciscus de Marchia that the impetus has not a permanent character. After this astounding golden period of the middle of the fourteenth century one starts to translate, comment, and alter all these theories and no real progress is made until the end of the sixteenth century. Two hundred years are in fact lost !

b) *Falling*

Already the best theory on the mechanics of falling objects has been cited in the previous paragraph. One can start the mechanics of falling objects with a text *liber de ratione ponderis* by Jordanus in the thirteenth century. He already gives a correct concept of acceleration by phrasing its consequences that the same distance is covered in ever shorter time lapses or that ever greater velocities occur in the same consecutive lapses of time. According to Jordanus the acceleration is due to the fact that the surrounding air is set into motion so that the resistance diminishes. The higher speed causes then a further diminution of the resistance and so on. Due to the fall the object becomes heavier and acquires a greater *impuls*. This *impuls* is thus a precursor of the *impetus* of Buridan. In the thirteenth century several authors comment on the mechanics of falling objects. Roger Bacon (1220-1292) introduces two forces : the constant *natural gravity* and the ever growing motor when an object comes closer towards its natural sphere. Other authors comment on the fact of the increasing speed in conjunction with the greater height covered by falling. The theory of falling objects attains its summit with Jean Buridan as cited above. Again after this very flourishing fourteenth century one has to wait until 1555 when Domingo de Soto proposes a correlation between the kinematics of the uniform accelerated motion as described by the Merton College and the kinematics of falling objects. The next step is forwarded by Galileo.

Conclusion

After the translation period a set of theories are produced mainly in Paris and Oxford in the middle of the fourteenth century. Due to the lack of experiments and appropriate mathematical techniques these spectacular advances come to a halt after only a few decennia. It takes then two hundred years to find in the Renaissance meaningful renovations in mechanics put forward by scientists such as Galileo, Simon Stevin, Huygens, Hooke and others. In 1687 modern mechanics is born when the *Philosophia naturalis Principia mathematica* is published by Isaac Newton.

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